

# PHASE TRANSITIONS AND VARIATIONAL ASPECTS OF MINIMAL HYPERSURFACES

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**Abstract:** In light of the connection between the theory of phase transitions and the critical points for the area functional we describe a variational construction of solutions to the elliptic *Allen-Cahn* equation

$$-\varepsilon\Delta u + \frac{W'(u)}{\varepsilon} = 0$$

in closed manifolds and the corresponding *limit interfaces*, that is the minimal hypersurfaces which arise as weak limits of the level sets of such solutions. This construction is inspired by recent developments on the min-max theory of minimal hypersurfaces and it extends well-known analogies regarding minimizers of the associated energy functional to more general variational solutions. This is joint work with M. Guaraco.

We also describe an upper bound for the stability index of limit interfaces which arise from solutions with uniformly bounded energy and index in terms of the Morse index of these solutions obtained by comparing the second inner variation of the energy functional to the second variation of the area.